Lecture 2: Descriptive Statistics

Center · Spread · Quantiles · Shape · ECDF · Anscombe · Simpson

You get a spreadsheet with 10,000 rows...

What do you look at first?

And what can fool you?

Before any model, any test, any estimation — **look at the data**.

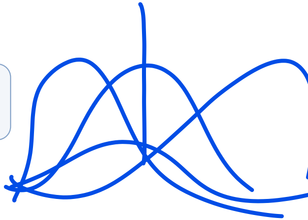
Goals of Descriptive Statistics

Summarize **center**, **spread**, and **shape**

Detect **outliers**, missing data, impossible values

Compare groups or time periods visually

Generate **hypotheses** before testing them



Descriptive $\neq$ inferential: we're describing *this sample*, not yet the population.

Measures of Center

Sample Mean

$\bar{X} = \frac{1}{n}\sum X_i$

Uses all data
Minimizes squared error

Sensitive to outliers

Sample Median

Middle value

Robust (50% breakdown)
Ignores magnitudes

Resists outliers

Mode

Most frequent value

Best for categorical
Can be non-unique

Minimizes 0–1 loss

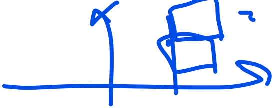
Trimmed Mean

Drop top/bottom k%

Compromise: mean $\leftrightarrow$ median

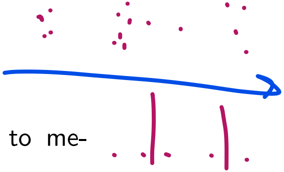
Tunable robustness

Measures of Spread



$E[(X - EX)]$

Measure	Formula	Properties
Variance S^2	$\frac{1}{n-1}\sum(X_i - \bar{X})^2$	Uses all data; $n-1$ = Bessel's correction (unbiased)
Std Dev S	$\boxed{\sqrt{S^2}}$	Same units as data
Range	max − min	Simple; extremely fragile
IQR	$Q_3 - Q_1$	Middle 50%; robust
MAD	med $\lvert X_i - \text{med} \rvert$	Most robust; companion to median

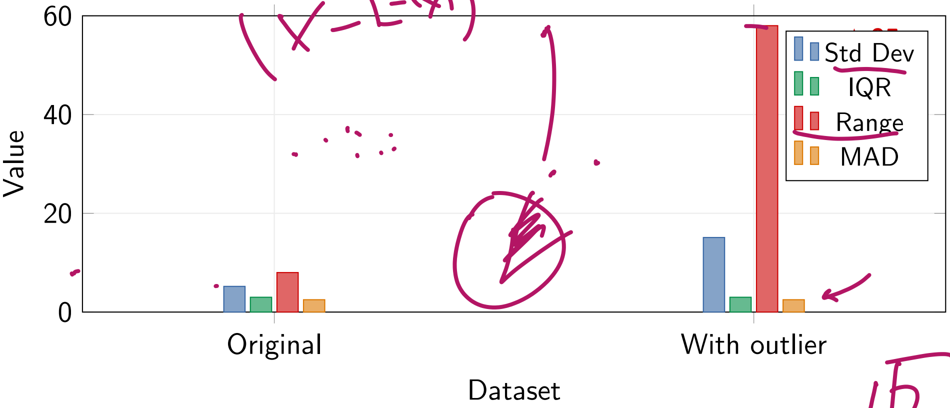


$\{ \lvert X_i - \text{med} \rvert : X_i \}$

6

Why $n - 1$? We used up one "degree of freedom" estimating $\bar{X}$. This makes S^2 **unbiased**: $\mathbb{E}[S^2] = \sigma^2$.

Robust vs Non-Robust: Visual



One outlier: Range explodes, Std Dev triples. IQR and MAD don't budge.

How a Histogram Is Built

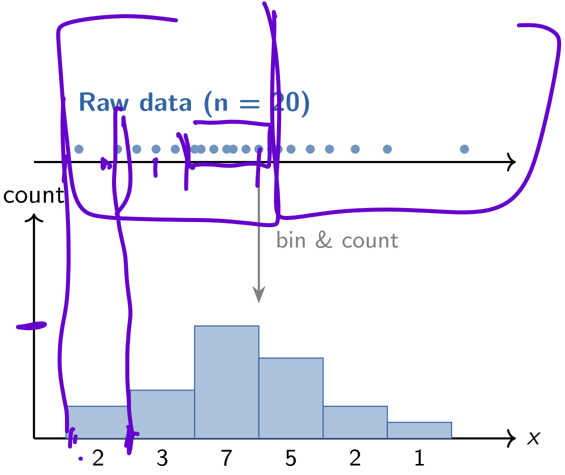
Recipe:

1. Choose **bins**: equal-width intervals covering the data range
2. Count observations in each bin
3. Draw bars — height = count (or density)

Density form:

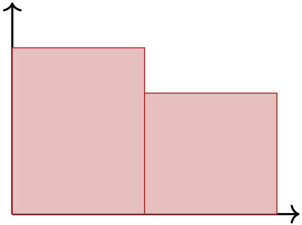
$$\text{height} = \frac{\text{count}}{n \times \text{bin width}}$$

so total area = 1 (comparable across bin widths).



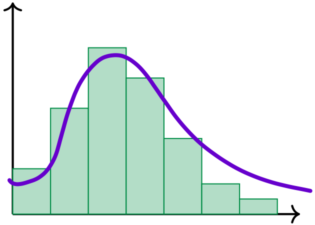
Bin Width Matters

Too few bins (2)



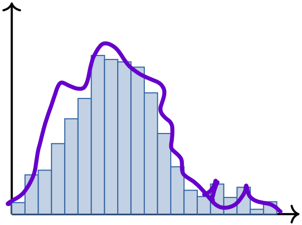
Hides all structure

Good bin width



Shape is clear

Too many bins (20)



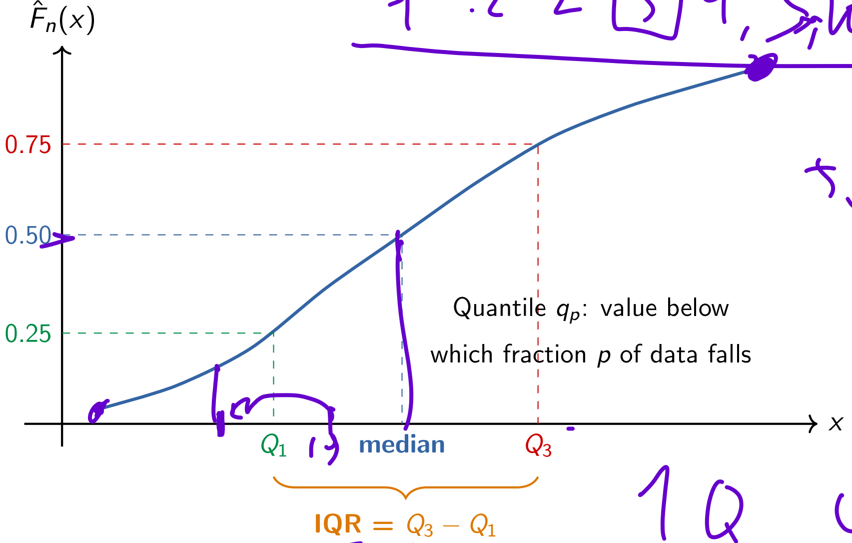
Too noisy

Common rules: Sturges ($k = 1 + \log_2 n$), Freedman–Diaconis ($h = 2 \cdot \mathrm{IQR} \cdot n^{-1/3}$).
In practice: try several and look.

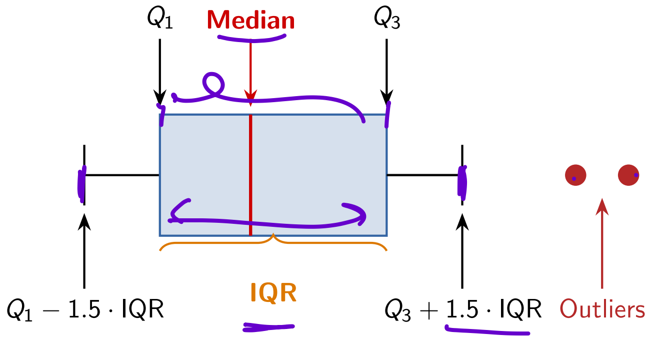
$1 + \log_2 8$ $2 \cdot$

Quantiles and Percentiles

1 2 2 $\boxed{3}$ 4, 5, 1000



Boxplot Anatomy



Five-number summary: min, Q_1, median, Q_3, max — the boxplot visualizes exactly this.

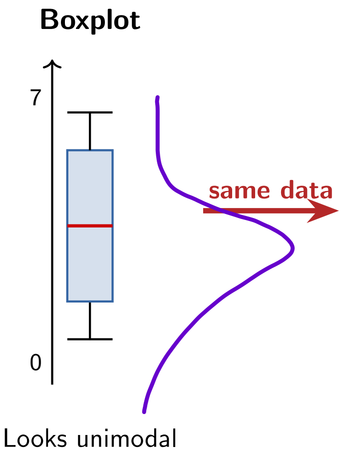
Strengths:

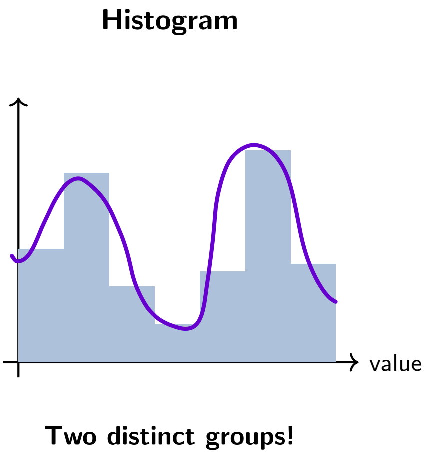
- Compact group comparison
- Shows center, spread, outliers

Weakness:

- Hides multimodality!
- Pair with histogram or violin

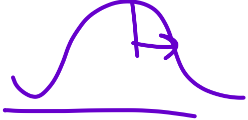
Boxplot Hides Bimodality



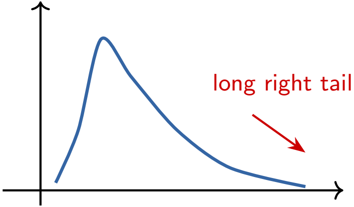


Always pair boxplots with histograms or violin plots.

Skewness: Measuring Asymmetry

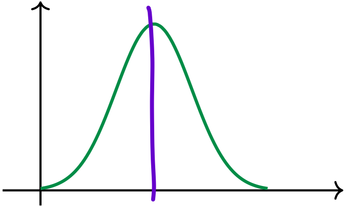


Positive Skew



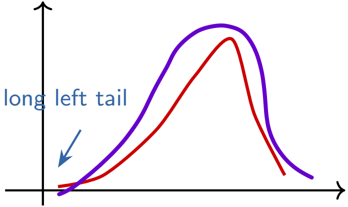
Income, house prices

Symmetric



Heights, measurement error

Negative Skew

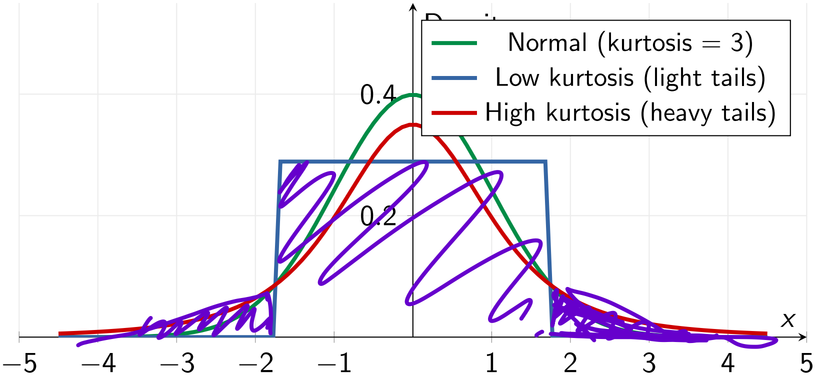


Exam scores near ceiling

$$\text{Skewness} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{X_i - \bar{X}}{S}\right)^3$$

$\Sigma X' (X-\bar{X})^2$

Kurtosis: Tail Heaviness

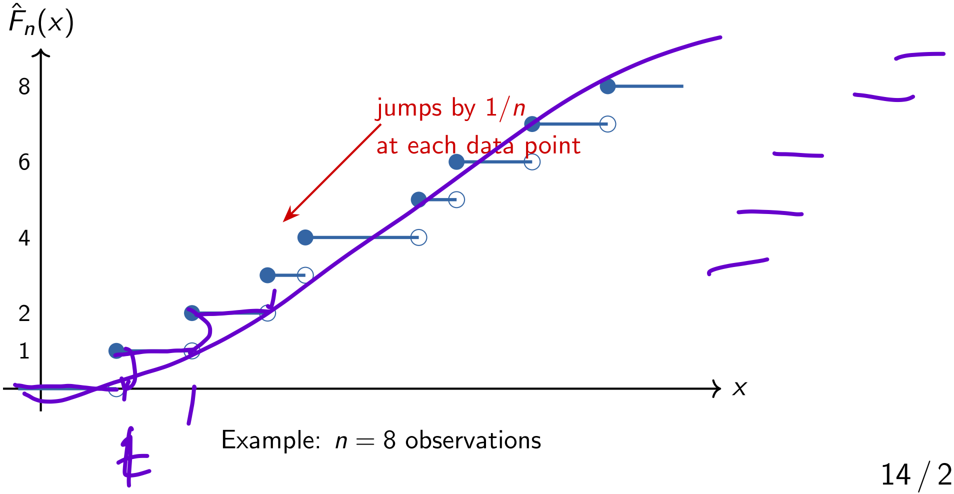


$$\text{Kurtosis} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{X_i - \bar{X}}{S}\right)^4 \qquad \text{Excess kurtosis} = \text{Kurt} - 3$$

Normal has kurtosis = 3 (excess = 0). Most software reports **excess kurtosis**.
Financial returns have high kurtosis — assuming normality underestimates risk.

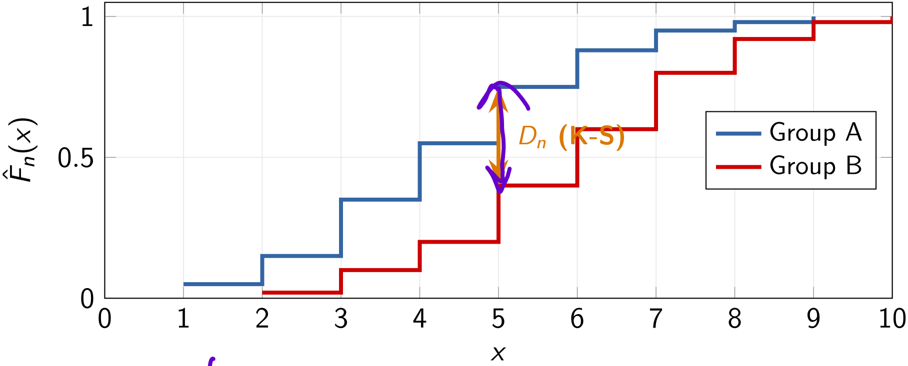
The Empirical CDF

$$\hat{F}_n(t) = \frac{1}{n}\#\{X_i \leq t\} = \frac{\text{number of observations} \leq t}{n}$$



Example: $n = 8$ observations

ECDF: Why It's Powerful



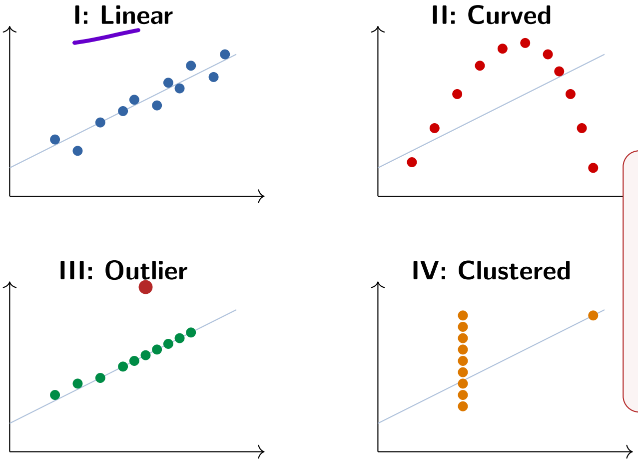
- No bin-width choice (unlike histograms) — the ECDF is **parameter-free**
- Biggest gap = **Kolmogorov–Smirnov statistic** D_n
- Glivenko–Cantelli: $\hat{F}_n \to F$ uniformly as $n \to \infty$

Choosing the Right Plot

What do I want to see?	Best plot
Distribution shape of one variable	**Histogram** / density plot
Compare distributions across groups	**Boxplot** / violin plot
Relationship between two variables	**Scatter plot**
Full distribution, no bin choices	**ECDF**
Change over time	**Line plot** / time series

Rule of thumb: always start with a histogram + scatter plot matrix. Then refine.

Anscombe's Quartet (1973)

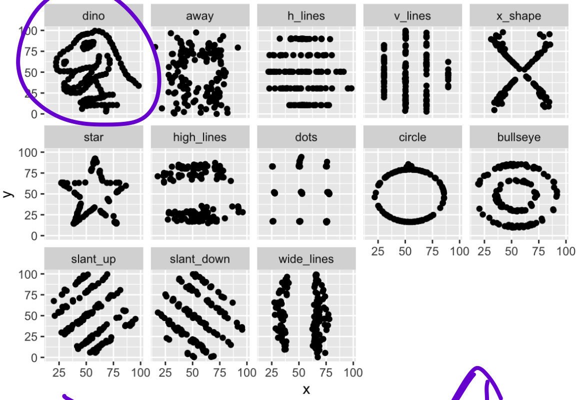


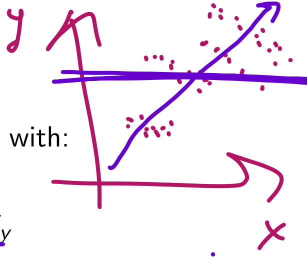
All four datasets:

$$\bar{x} = 9,\ \bar{y} \approx 7.5$$
$$S_x^2 = 11,\ S_y^2 \approx 4.13$$
$$r \approx 0.816$$
$$\hat{y} = 3 + 0.5x$$

Identical statistics.
Wildly different data.

The Datasaurus Dozen (2017)



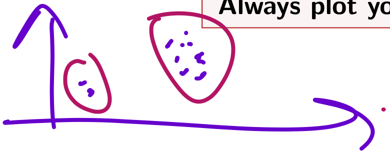


13 datasets, all with:

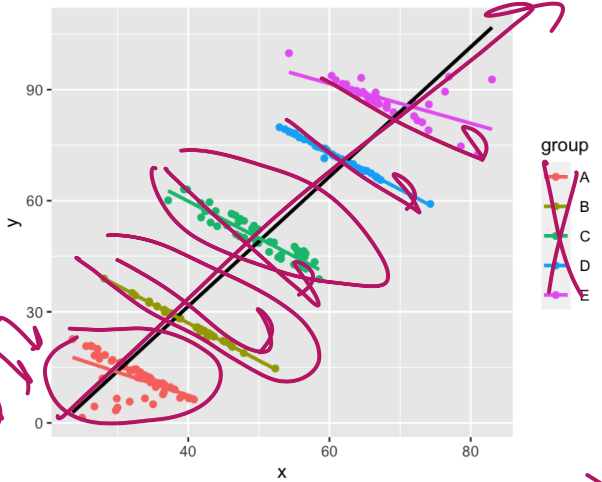
- Same $\bar{x}$, $\bar{y}$
- Same S_x, S_y
- Same correlation r

Yet shapes include a **dinosaur**, a star, parallel lines, a circle...

Never trust summary statistics alone. Always plot your data.



Simpson's Paradox



Each colored group trends **down**, yet the aggregate trend goes **up**.
How? The groups have **different sizes and positions**.

Simpson's Paradox: UC Berkeley Admissions (1973)

12,763 applicants to UC Berkeley graduate programs.

Aggregate data:

	Applied	Admitted
Men	8,442	44%
Women	4,321	35%

9 percentage points gap!
Lawsuit filed for gender bias.

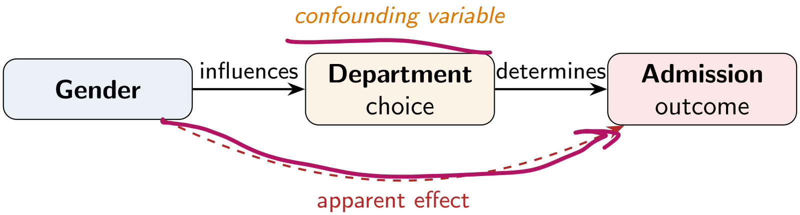
By department:

	Rate	Women	Men
Dept A	easy	82%	62%
Dept B	easy	68%	63%
Dept C	hard	34%	35%
Dept D	hard	7%	6%

Women admitted at equal or **higher** rates in each dept!

How is this possible?

Simpson's Paradox: Why It Happens



The key: Women disproportionately applied to **hard** departments (low acceptance for everyone). Men disproportionately applied to **easy** departments.

When you aggregate, the **different weights** reverse the trend:

- Women: ∼80% applied to hard depts → low overall rate
- Men: ∼80% applied to easy depts → high overall rate

> **General lesson:** a trend in every subgroup can **reverse** when subgroups are combined.
> Always ask: *is there a hidden variable that changes the group sizes?*

Homework

1. Go over the **Data Visualization** topic:

   `https://hayktarkhanyan.github.io/python_math_ml_course/python_libs/06_data_viz.html`

2. Pick a dataset (e.g. from **Kaggle** or `http://armstat.am/`) and **explore** it:

   compute summary statistics, build histograms, boxplots, scatter plots, ECDF

3. Come up with your own examples of:
   - **Survivorship bias**
   - **Simpson's paradox**

Questions?