Setup

A thermometer measures temperature with known measurement noise. Each reading is:

$$\boxed{X_i \mid \mu \sim N(\mu,\ \sigma^2)}, \qquad \sigma = 2\ ^\circ\mathrm{C}\ \text{(known)}$$

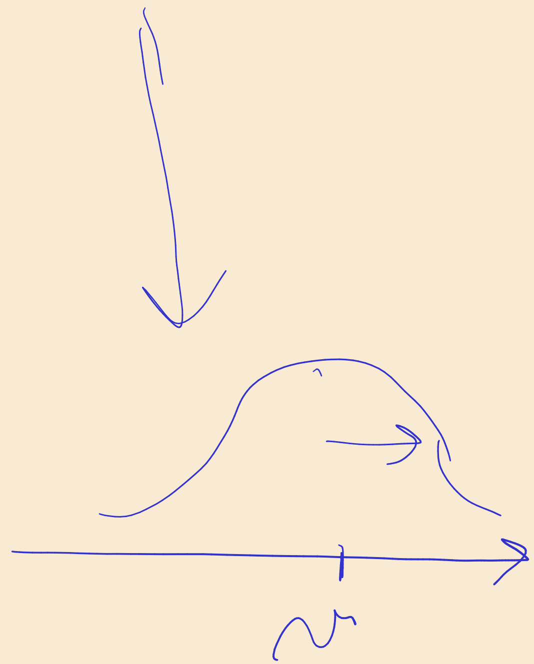
You take $n = 5$ independent readings:

$$\boxed{x_1 = 21.3, \quad x_2 = 19.8, \quad x_3 = 22.1, \quad x_4 = 20.5, \quad x_5 = 23.0}$$

The manufacturer says these sensors are calibrated around 20 °C, so you use a Gaussian prior:

$$\boxed{\mu \sim N(m,\ \tau^2) = \boxed{N(20,\ 3^2)}}$$

Goal: Estimate the true temperature μ using (a) MLE and (b) MAP.



$X_i \mid \sim$

$\boxed{\text{MLE}}$ MoM

$f(x_1, x_2 \ldots)$

$[0, d)$

$\max(x_1, x_2, \ldots)$

$\hat{\theta} = \frac{1}{n}\sum x_i$

Bias

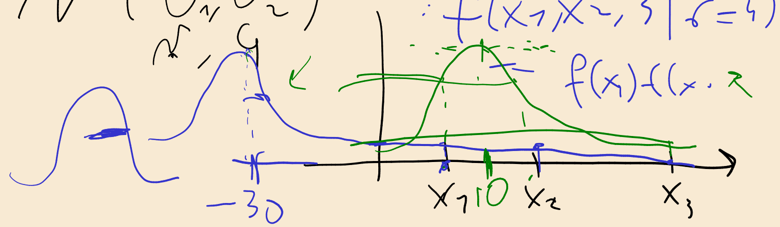
cons

MLE $L(d)$

$f(x_1, x_2, 3 \mid \sigma^2 = 4)$

$f(x_1) f(x_2 \ldots$

$X_1, X_2, X_3 \sim N(\theta_1, \theta_2)$



$$L(\mu) = \prod_{i=1}^{n} f(x_i|\mu) = \qquad \boxed{f(x_1, x_2, \ldots | \mu)}$$

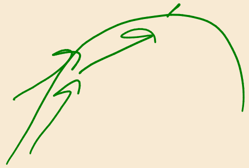
$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right) \qquad f_1(x_1)\cdot g(x_2)\cdots$$

$$\ell(\mu) = \cancel{-\frac{n}{2}\ln(2\pi\sigma^2)} - \frac{1}{2\sigma^2}\boxed{\sum_{i=1}^{n}(x_i-\mu)^2}$$

$$\left(\sum x_i - n\mu\right) = 0$$

$$\mu = \frac{1}{n}\sum x_i$$

$f(\mu)$



$\frac{1}{5}$

ML $\quad N(21.34, 4)$

21.34

$$\boxed{x_1, x_2, \ldots x_5 \sim N(\mu, 4)}$$

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