🧀 🧀 29 PMF and CDF for Two Coin Flips

A fair coin is tossed twice. Let X be the number of observed heads.

- a. Find the PMF of X.
- b. Find the CDF of X.
- c. Plot the PMF and the CDF.

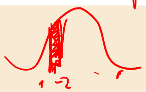
🧀 🧀 30 Exponential PDF and CDF

Let X be a continuous random variable with PDF

f_X(x) = { c e^{-x}, x ≥ 0; 0, otherwise }, where c > 0.

- a. Find the value of c.
- b. Find the CDF F_X(x).
- c. Find P(1 < X < 3).

ce^{-x}



PMF

$p(H) = \frac{1}{2}$

$p(T) = \frac{1}{2}$

$X: \Omega \to [0,1]$

$\Omega = \{HH, TT, HT, TH\}$

CDF

$P(X=0) = \frac{1}{4}$

$P(X=1) = \frac{2}{4}$

$P(X=2) = \frac{1}{4}$

$P(X=3) = 0$

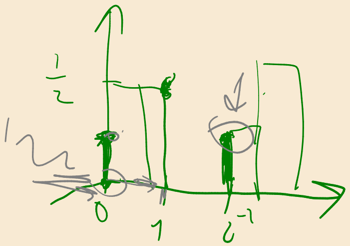
$$\boxed{\begin{array}{l} 0 \to \frac{1}{4} \\ 1 \to \frac{2}{4} = \frac{1}{2} \\ 2 \to \frac{1}{4} \end{array}}$$

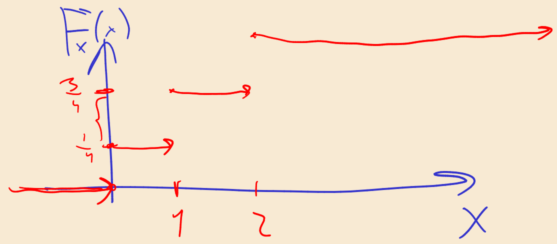
$F_X(x) = P(X \le x)$

$F_X(1) = P(X \le 1)$

$\frac{1}{4} + \frac{1}{2}$

$$F_X(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{4}, & 0 \le x < 1 \\ \frac{1}{4} + \frac{1}{2}, & x < 2 \\ \frac{1}{4} + \frac{1}{2} + \frac{1}{4}, & x \ge 2 \end{cases}$$





CDF

$c > 0$

$$f_X(x) = \begin{cases} ce^{-x}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

$(e^{x})' = e^{x}$

$(e^{-x})' = -e^{-x} \cdot (-x)'$

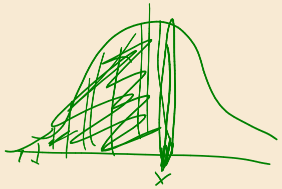
$$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} ce^{-x}\,dx = c\int_{0}^{\infty} e^{-x}\,dx = -ce^{-x}\Big|_{0}^{\infty} = 1$$

$$= -c\left(\frac{1}{e^{\infty}} - e^{0}\right) = 1$$

$c = 1$

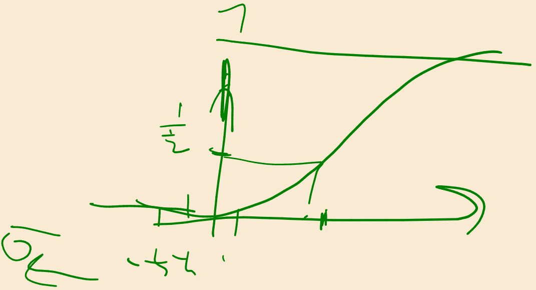
$F_X(x) = P(X \le x)$

$x < 0 \Rightarrow F_X(x) = 0$

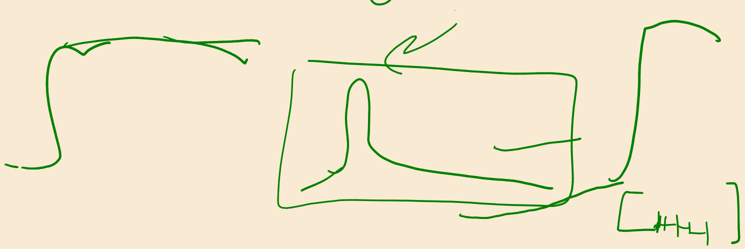


$F_X(x) = \int_0^x e^{-t}\,dt = -e^{-t}\Big|_0^x =$

$= -(e^{-x} - e^{-0}) = \boxed{1 - e^{-x}}$

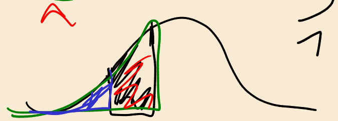


$-5, -4, -3$

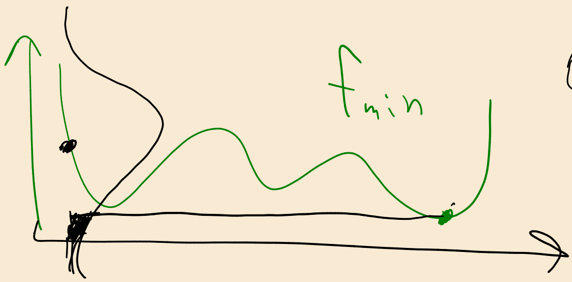


$P(1 < X < 3) =$

$= \int_1^3 e^{-t} = F_X(3) - F_X(1) = 1 - e^{-3} - (1 - e^{-1}) = e^{-1} - e^{-3}$



$1 - 3 \quad -3$



$0,$

$P(1 < X < 3)$

Problem: Independence of Two Random Variables

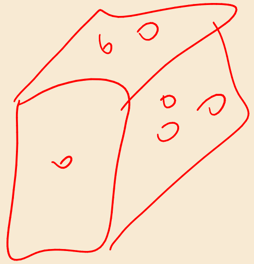
A fair six-sided die is rolled once.

Define the random variables:

- $X = 1$ if the outcome is **even**, and $X = 0$ if the outcome is **odd**.
- $Y = 1$ if the outcome is **greater than 3**, and $Y = 0$ otherwise.

Tasks

1. Find the joint probability table of (X, Y).
2. Find the marginal distributions of X and Y.
3. Determine whether X and Y are independent.



$X = 1$, even

$0 \to$ odd,

$Y = 1 > 3$

$= 0$,

(X, Y) joint

4, 5, 6

	1	2	3	4	5	6
X	0	1	0	1	0	1
Y	0	0	0	1	1	1

$1/6$ $\quad$ $1/6$

$X \backslash Y$	0	1	
0	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$
1	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
	$\frac{1}{2}$	$\frac{1}{2}$	

$\frac{1}{3} + \frac{1}{6}$

$$P(X=1, Y=1) = \frac{1}{3} \neq$$

$$= P(X=1) \cdot P(Y=1) = \frac{1}{2} \cdot \frac{1}{2}$$

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{6} + \frac{1}{3} = 1$$

P

10.000