01 Two Dice Probability

Suppose we roll two fair dice. What is the probability of getting:

a. 2 on each of them
b. at least one 1
c. exactly one 1
d. one 1 and one 4
e. 1 on the first die and 4 on the second die

02 Coin Tosses - Odd Heads

A fair coin is tossed 5 times. What is the probability of getting an odd number of heads?

Can you do the exercise without much computations?

03 Queen or Heart

A standard deck of 52 playing cards is shuffled. What is the probability of drawing either a queen or a heart?

5×5

$1 - \frac{25}{36}$

6×6 $(2,2)$

$\frac{\#\text{succ}}{\#\text{all}} = \frac{1}{36}$

$\frac{1}{6} \cdot \frac{5}{6} + \frac{5}{6} \cdot \frac{1}{6} = 2 \cdot \frac{5}{36} = \frac{10}{36} = \frac{5}{18}$

$\frac{1}{2}\left(\frac{1}{2}\right)^4$

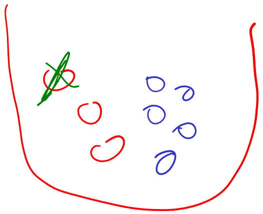
🧀🧀 04 An Urn

An urn contains 3 red balls and 5 blue balls. Two balls are drawn without replacement. What is the probability that both balls are red?

🧀 05 Colored Pencils

There are 2 red, 5 blue and 6 yellow pencils (total: 13). Two pencils are drawn randomly. Find the probability that both are:

a. red
b. of the same color
c. of different colors
d. not yellow
e. not green



$$\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

$$\frac{C_3^2}{C_8^2}$$

$$\frac{\frac{3 \cdot 2}{2}}{\frac{8 \cdot 7}{2}}$$

$$\frac{3}{4 \cdot 7} = \frac{3}{28}$$

🧀🧀 06 Reading Books

There are 15 books: 5 in Armenian, 10 in French. Ruben cannot read French. If he randomly takes 3 books, what is the probability that he can read at least one?

I $\frac{3}{8}$ II $\frac{2}{7}$

$$\frac{3}{8} \cdot \frac{2}{7} = \frac{3}{28}$$

🧀🧀 07 Baby-Mother Matching

Three babies are given a weekly health check at a clinic, and then returned randomly to their mothers. What is the probability that at least one baby goes to the right mother?

0.1

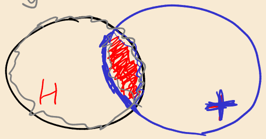
99%

5%

$$P(H|+) = \frac{P(+|H)\cdot P(H)}{P(+)}$$

$$P(H|+)\cdot P(+) = P(+|H)\cdot P(H)$$

$$P(H\cap +) = P(+\cap H)$$



$H\cap +$

🧀 🧀 18 Medical Test (Bayes' Theorem)

A disease affects 1 in 1,000 people (0.1%). A test has: true positive rate 99% (if diseased), false positive rate 5% (if healthy). If a person tests positive, what is the probability they actually have the disease?

$$P(H|+) = \frac{P(+|H)\cdot P(H)}{P(+)}$$

$P(H) = 0.001$ $\quad P(+|H) = 0.99$

$$P(+) = P(+\cap H) + P(+\cap H^c)$$

$P(+|H)\cdot P(H)$ $\quad P(+|H^c)\cdot P(H^c)$

$\boxed{0.001}$

$0.99\cdot 0.001$ $\quad 0.05$ $\quad 0.999$

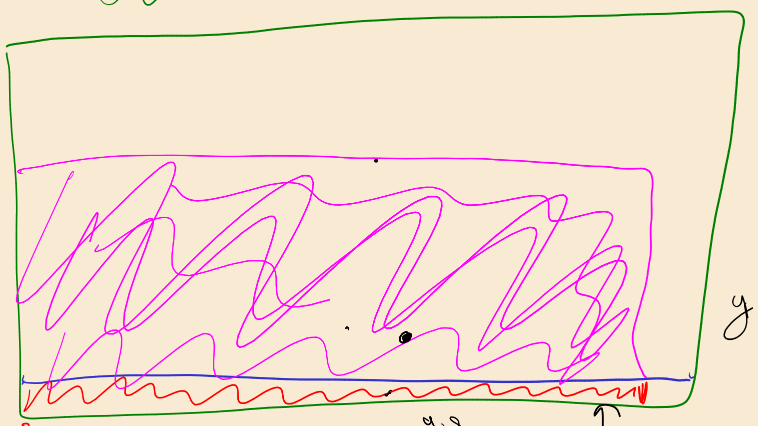
2% $\quad 0.001$ $\quad 0.02$

100.000

1/1000

2h funkty - 100

99.900



2,5

y

0.05 ≈ 5000

4995

99

9945000 ≈ 2% 99%

Type 1/2

y	ŷ
1	0
0	1

1

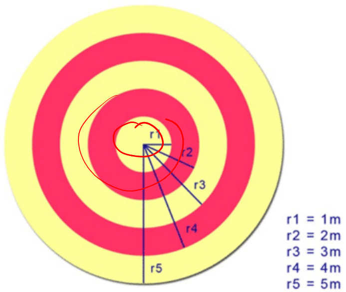
1%

5%

🧀 🧀 09 Dart Throwing

A dart is thrown at a circular target with concentric circles. Circle 1 (innermost) has radius 1m, and each subsequent radius increases by 1m. Find the probability that the dart lands in:

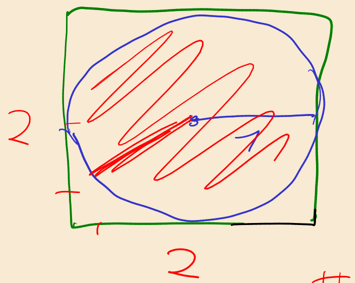
a. circle 1
b. a red circle
c. a yellow circle



dart_throwing

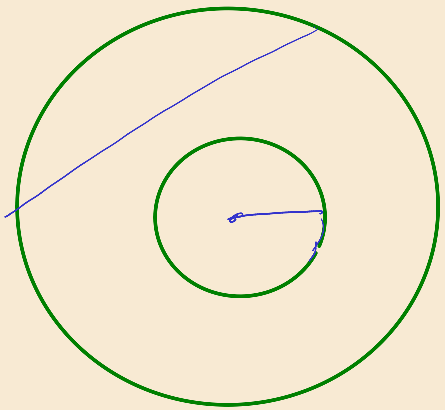
🧀 🧀 10 Computing Pi

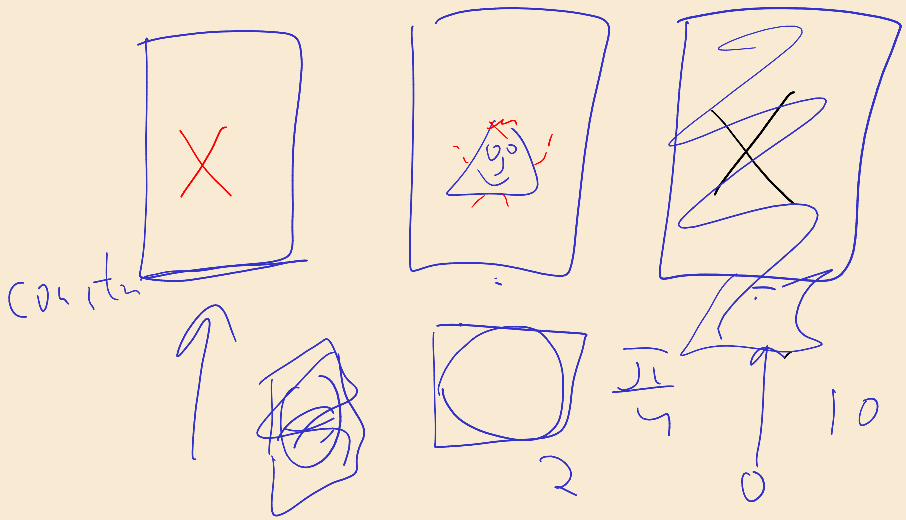
What is the probability that a randomly chosen point inside a square of side length 2 falls within the inscribed circle of radius 1?



$\pi 1^2 = \pi$

$\frac{\#\bigcirc}{\#\square} = \frac{\pi}{4}$





🧀 🧀 09 Birthday Paradox

In a group of n people, what is the probability that at least two share the same birthday (assume 365 days and ignore leap years)? Approximately how many people are needed for this probability to exceed 50%?

23

365

$$1 - \left(1 \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdots\right)^{32} \quad \frac{23}{365}$$

23 50%

n^2 $\frac{23 \cdot 22}{2} = 23 \cdot 11 = 253$

253

C_{23}^{2}

b, d

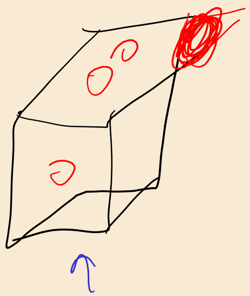
$3 \to \frac{3 \cdot 2}{2} \to 3$

$23 \to 253$

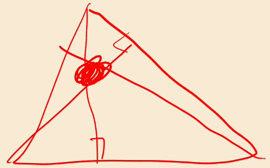
🧀 🧀 25 Զառեր

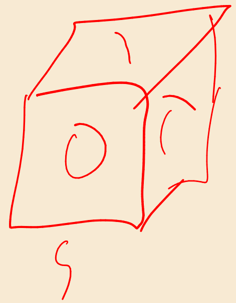
Ունենք երկու զառ, որոնցից մեկը՝ սովորական, 1-6 թվերով, իսկ մյուսը՝ դատարկ, առանց թվերի: Ի՞նչ թվեր գրենք 2-րդ զառի վրա, որպեսզի երկու զառերը միաժամանակ նետելիս դրանց թվերի գումարի հնարավոր արժեքները լինեն 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

ընդ որում բոլորը **հավասար հավանականությամբ**:



12

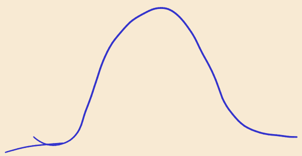




q1 + q2

1 - 12

$\boxed{1-12}$



12, 12

1, 0

6

6, 6

(1, 1)

36

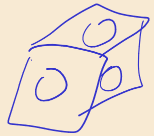
$\frac{1}{12}$

36

x = 3

$\frac{x}{36} = \frac{1}{12}$

0 1,0



1,0
1,0
1,0
1,1
:
2,4
2,6

2,0
2,0

7

1 36
2 3

12

12 x3 6,6

7	12
0	8

000666

7,3 → 10

7,6

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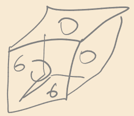
1 — 12 36



111 222 333 .... 12 12 12

1,0 1+-1=0

2,
3,
4,
5,

36

0

$\frac{1}{12} = \frac{3}{6}$

$\frac{x}{36} = \frac{1}{12}$

3,9

6,9 → 15

12 6,6